Error Probability Bounds for M-ary Relay Trees

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***Abstract*—We study the detection error probabilities associated with an M-ary relay tree, where the leaves of the tree correspond to identical and independent sensors. Only these leaves are sensors. The root of the tree represents a fusion center that makes the overall detection decision. Each of the other nodes in the tree is a relay node that combines M summarized messages from its immediate child nodes to form a single output message using the majority dominance rule. We derive tight upper and lower bounds for the Type I and II error probabilities at the fusion center as explicit functions of the number of sensors in the case of binary message alphabets. These bounds characterize how fast the error probabilities converge to 0 with respect to the number of sensors.**

***Index Terms*—M-ary relay tree, distributed detection, decay rate.**

I. Introduction

Consider the binary hypothesis testing problem in a *distributed detection* network [1]. Each sensor makes a measurement and summarizes its measurement into a smaller message, and then sends it to the fusion center, possibly via intermediate relay nodes. The fusion center then makes a final decision between two hypotheses. Several distributed detection configurations have been investigated. In the well-studied parallel architecture [1]–[3], each sensor directly sends its compressed information to the fusion center. With the assumption of conditional independence of the sensors, the error probability in the parallel architecture decays exponentially. Another configuration considered is the tandem network [4]–[6], in which each non-leaf node aggregates its own measurement with the message it has received from its immediate child node at one level down, which is then transmitted to its parent node at the next level up. The error probability in this case decays sub-exponentially [6] with respect to the number of sensors.

The detection performance of bounded-height tree networks is discussed in [7] and [8]. Even though the error probability in the parallel configuration decays more quickly than in the tree network, in a tree network we usually have shorter-range communications via aggregations through local nodes, which could significantly reduce the communication resources. In the tree network, measurements are summarized by leaf sensor nodes into smaller messages and sent to their parent nodes, each of which fuses all the messages it receives with its own measurement (if any) and then forwards the new message to its parent node at the next level. This process takes place throughout the tree, culminating at the fusion center where an overall decision is made. For bounded-height tree networks where leaf nodes dominate, the optimal error exponent is the same as that of the parallel configuration [7].

The detection performance for trees with unbounded height is still largely unexplored. The balanced binary relay tree was addressed in [9], providing the convergence of the Type I (also known as false alarm) and Type II (also known as missed detection) error probabilities. In [10], we derived tight upper and lower bounds for the total error probability at the fusion center, which show that the decay exponent of the total error probability is $\sqrt{N}$. In [11] and [12], we studied the case where sensors fail with certain probabilities and the case where the communication links fail with certain probabilities. We showed that in the sensor failure case, the convergence of the total error probability at the fusion center is sub-exponential with the same exponent $\sqrt{N}$. In the link failure case, if the link failure probabilities decay sufficiently fast as we move up to the fusion center, then the total error probability decays to 0 with the same exponent $\sqrt{N}$.

In [13], a similar problem was considered in an M-ary relay tree configuration, where each node with the exception of the sensors has M child nodes. Notice that balanced binary relay trees are simply special cases of M-ary relay trees. To describe the result in [13], let P_N be the total error probability at the fusion center and suppose that each sensor and relay node only transmit binary messages upward to a node at the next level. Then, it is shown in [13] that with any combination of fusion rules, the decay exponent is upper bounded:

$$\log_2 P_N^{-1} = O(N^{\log_M \frac{(M+1)}{2}}).$$

The case where the relay nodes and the fusion center use the majority dominance rule (with random tie-breaking) to combine messages was also considered in [13], in which case the decay rate of the total error probability is almost optimal. More precisely,

$$\log_2 P_N^{-1} = \Omega(N^{\log_M \lfloor \frac{(M+1)}{2} \rfloor}).$$

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Therefore, in the case where M is odd, the majority dominance rule achieves the optimal decay exponent. In the case where M is even, there is a gap between the two bounds. This gap is evident in the case of balanced binary relay tree ($M = 2$), where it is easy to show that the Type I and II error probabilities do not change after fusion with the majority dominance rule. This shows that the lower bound above is tight. On the other hand, we have shown that the decay exponent is $\sqrt{N}$ when using unit-threshold likelihood-ratio test [10].

In this paper, we use the approach in [10] to study the detection performance of M-ary relay trees. In contrast to the results in [13], which only address the asymptotic regime, we derive tight upper and lower bounds for the Type I and II error probabilities at the fusion center as explicit functions of N. We show that the majority dominance rule is essentially sub-optimal in the case where M is even. Specifically, our result shows that for all M,

$$\log_2 P_N^{-1} = O(N^{\log_M \lfloor \frac{(M+1)}{2} \rfloor}),$$

a result not present in [13].

II. Problem Formulation

We consider the problem of binary hypothesis testing between H_0 and H_1 in an M-ary relay tree. Let $\mathbb{P}_0$ and $\mathbb{P}_1$ be the probability measures associated with the binary hypotheses. As shown in Fig. 1, leaf nodes are sensors undertaking initial and independent measurements of the same event. Only the leaves are sensors making measurements in this tree architecture. These measurements are compressed into binary messages and forwarded to the parent nodes at the next level. Each non-leaf node with the exception of the root, the fusion center, is a relay node, which combines M binary messages into one new binary message and forwards the new binary message to its parent node. This process takes place at each node, culminating at the fusion center at which the final decision is made based on the information received. The height of the tree is $\log_M N$, which grows as the number of sensors increases. Evidently, for $M = 2$, the structure is simply a balanced binary relay tree, which is the worst-case scenario in the sense of largest tree height among M-ary relay trees.

We assume that all sensors are independent given each hypothesis, and that all sensors have identical Type I error probability (denoted by α_0) and identical Type II error probability (denoted by β_0). We apply the majority dominance rule as the fusion rule at the relay nodes and at the fusion center. We answer the following questions about the Type I and II error probabilities:

- How do they change as we move upward in the tree?
- What are their explicit forms as explicit functions of N?
- Do they converge to 0 at the fusion center?
- How fast will they converge with respect to N?

III. Error Probability Bounds

We divide our analysis into two cases: M is an odd integer (oddary tree) and M is an even integer (evenary tree). In each

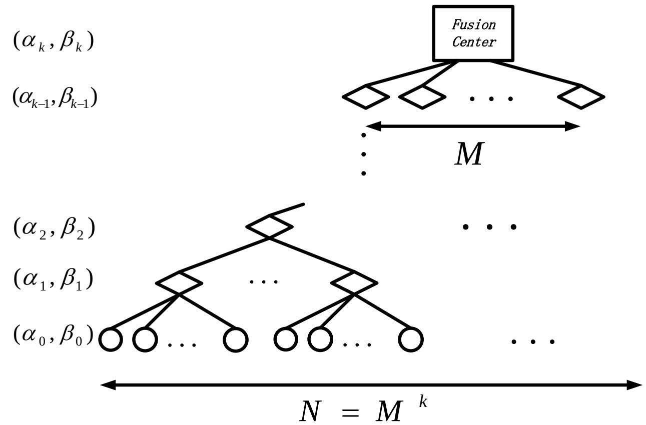

Fig. 1. An M-ary relay tree with height k. Circles represent sensors making initial measurements. Diamonds represent relay nodes which fuse M binary messages. The rectangle at the root represents the fusion center making an overall decision.

case, we first derive the recursion of the Type I and II error probabilities and show that all nodes at level k have the same Type I and II error probabilities (α_k, β_k).. Then we study the step-wise reduction of each kind of error probability after fusion with majority dominance rule. From these we provide upper and lower bounds for the Type I and II error probability at the fusion center. We then derive upper and lower bounds for the total error probability at the fusion center.

A. Oddary Tree

Suppose that u_o is the output binary message after fusing M input binary messages $\mathbf{u}_i = \{u_1, u_2, \ldots, u_M\}$, where $u_t \in \{0, 1\}$ for all t. The majority dominance rule, when M is odd, is simply:

$$u_o := \begin{cases} 1, & \text{if } u_1 + u_2 + \ldots + u_M \geq M/2, \\ 0, & \text{if } u_1 + u_2 + \ldots + u_M \leq M/2. \end{cases}$$

Assume binary messages $\{u_i\}_{i=1}^M$ have identical Type I error probability α and identical Type II error probability β. Then, the Type I and II error probability pair (α', β') for the binary message u_o is given by:

$$\begin{aligned} \alpha' = \mathbb{P}_0(u_o = 1) &= \prod_{i=1}^{M} \mathbb{P}_0(u_i = 1) \\ &+ \binom{M}{1} \mathbb{P}_0(u_t = 0) \prod_{i=1}^{M-1} \mathbb{P}_0(u_i = 1) + \ldots \\ &+ \binom{M}{(M-1)/2} \prod_{i=1}^{(M+1)/2} \mathbb{P}_0(u_i = 1) \prod_{t=1}^{(M-1)/2} \mathbb{P}_0(u_t = 0) \\ &= f(\alpha) := \alpha^M + \binom{M}{1} \alpha^{M-1}(1-\alpha) + \ldots \\ &+ \binom{M}{(M-1)/2} \alpha^{(M+1)/2} (1-\alpha)^{(M-1)/2} \end{aligned}$$

and

$$\beta' = f(\beta) = \beta^M + \binom{M}{1}\beta^{M-1}(1-\beta) + \ldots$$
$$+ \binom{M}{(M-1)/2}\beta^{(M+1)/2}(1-\beta)^{(M-1)/2}.$$

As all sensors have the same error probability pair (α_0, β_0), all relay nodes at level 1 will have the same error probability pair $(\alpha_1, \beta_1) = (f(\alpha_0), f(\beta_0))$. By recursion, we have

$$(\alpha_{k+1}, \beta_{k+1}) = (f(\alpha_k), f(\beta_k)), \qquad k = 0, 1, \ldots,$$

where (α_k, β_k) is the error probability pair of nodes at the k-th level of the tree. Since the recursions for α_k and β_k are the same,it suffices to consider only the Type I error probability α_k in studying the decay speed. Next we will analyze the step-wise shrinkage of the Type I error probability after each fusion step. This analysis will in turn provide upper and lower bounds for the Type I error probability at the fusion center.

Proposition 1: Consider an M-ary relay tree, where M is odd. Suppose that we apply majority dominance rule as the fusion rule. Then,

$$1 \leq \frac{\alpha_{k+1}}{\alpha_k^{(M+1)/2}} \leq 2^{M-1}.$$

Proof: Consider the ratio of α_{k+1} and $\alpha_k^{(M+1)/2}$:

$$\frac{\alpha_{k+1}}{\alpha_k^{(M+1)/2}} = \alpha_k^{(M-1)/2} + \binom{M}{1}\alpha_k^{(M-3)/2}(1-\alpha_k)$$
$$+ \ldots + \binom{M}{(M-1)/2}(1-\alpha_k)^{(M-1)/2}.$$

First, we derive the lower bound of the ratio. We know that

$$\begin{aligned} 1 &= (\alpha_k + 1 - \alpha_k)^{(M-1)/2} \\ &= \alpha_k^{(M-1)/2} + \binom{(M-1)/2}{1}\alpha_k^{(M-3)/2}(1-\alpha_k) \\ &+ \ldots + \binom{(M-1)/2}{(M-1)/2}(1-\alpha_k)^{(M-1)/2}. \end{aligned}$$

Moreover, it is easy to see that

$$\binom{M}{k} \geq \binom{(M-1)/2}{k}$$

for all $k = 1, 2, \ldots, (M-1)/2$. In consequence, we have

$$\frac{\alpha_{k+1}}{\alpha_k^{(M+1)/2}} \geq 1.$$

Next we derive the upper bound of the ratio. Since $\alpha_k < 1$, we have

$$\frac{\alpha_{k+1}}{\alpha_k^{(M+1)/2}} \leq 1 + \binom{M}{1} + \ldots + \binom{M}{(M-1)/2} = 2^{M-1}.$$

■

Using the above proposition, we now derive upper and lower bounds for $\log_2 \alpha_k^{-1}$.

Theorem 1: Consider an M-ary relay tree, where M is an odd integer. Let $\lambda_M = (M+1)/2$. We have

$$\lambda_M^k(\log_2 \alpha_0^{-1} - (M-1)) \leq \log_2 \alpha_k^{-1} \leq \lambda_M^k \log_2 \alpha_0^{-1}.$$

Proof: From the inequalities in Proposition 1, we have

$$\alpha_{k+1} = c_k \alpha_k^{(M+1)/2} = c_k \alpha_k^{\lambda_M},$$

where $c_k \in [1, 2^{M-1}]$. From these we obtain

$$\alpha_k = c_{k-1} c_{k-2}^{\lambda_M} \ldots c_0^{\lambda_M{}^{k-1}} \alpha_0^{\lambda_M{}^k},$$

where $c_i \in [1, 2^{M-1}]$ for all i, and

$$\begin{aligned} \log_2 \alpha_k^{-1} = &- \log_2 c_{k-1} - \lambda_M \log_2 c_{k-2} - \ldots \\ &- \lambda_M^{k-1} \log_2 c_0 + \lambda_M^k \log_2 \alpha_0^{-1}. \end{aligned}$$

Since $\log_2 c_i \in [0, (M-1)]$, we have

$$\log_2 \alpha_k^{-1} \leq \lambda_M^k \log_2 \alpha_0^{-1}.$$

Moreover, we obtain

$$\begin{aligned} \log_2 \alpha_k^{-1} \geq &- (M-1) - \lambda_M(M-1) - \ldots \\ &- \lambda_M^{k-1}(M-1) + \lambda_M^k \log_2 \alpha_0^{-1} \\ \geq &\ \lambda_M^k(\log_2 \alpha_0^{-1} - (M-1)). \end{aligned}$$

■

In contrast to the result in [13], which only focuses on the asymptotic regime, our result holds for all finite k. In addition, the result in [13] deals with the total error probability at the fusion center. But our approach provides bounds for both Type I and II error probabilities.

Corollary 1: Let $P_{F,N}$ be the Type I error probability at the fusion center of an M-ary relay tree, where M is odd. We have

$$N^{\log_M \lambda_M}(\log_2 \alpha_0^{-1} - (M-1)) \leq$$
$$\log_2 P_{F,N}^{-1} \leq N^{\log_M \lambda_M} \log_2 \alpha_0^{-1}.$$

B. Evenary Tree

We now study the case where M is even and derive upper and lower bounds for Type I error probability. We still use the majority dominance rule (with random tie-breaking) as the fusion rule at the relay nodes and at the fusion center. The majority dominance rule in this case is:

$$u_o := \begin{cases} 1, & \text{if } u_1 + u_2 + \ldots + u_M > M/2, \\ 0 \text{ w.p. } P_b, & \text{if } u_1 + u_2 + \ldots + u_M = M/2, \\ 1 \text{ w.p. } 1 - P_b, & \text{if } u_1 + u_2 + \ldots + u_M = M/2, \\ 0, & \text{if } u_1 + u_2 + \ldots + u_M < M/2, \end{cases}$$

where P_b denotes the Bernoulli parameter for the tie-breaking case. For simplicity, we assume that tie-breaking

is fifty-fifty in this paper; i.e., $P_b = 1/2$. In this case, the recursions for the Type I and II error probabilities are:

$$\begin{aligned}\alpha' &= \mathbb{P}_0(u_o = 1) = \prod_{i=1}^{M} \mathbb{P}_0(u_i = 1) \\ &+ \binom{M}{1} \mathbb{P}_0(u_t = 0) \prod_{i=1}^{M-1} \mathbb{P}_0(u_i = 1) + \dots \\ &+ \frac{1}{2}\binom{M}{M/2} \prod_{i=1}^{M/2} \mathbb{P}_0(u_i = 1) \prod_{t=1}^{M/2} \mathbb{P}_0(u_t = 0) \\ &= g(\alpha) := \alpha^M + \binom{M}{1}\alpha^{M-1}(1-\alpha) + \dots \\ &+ \frac{1}{2}\binom{M}{M/2}\alpha^{M/2}(1-\alpha)^{M/2}\end{aligned}$$

and

$$\begin{aligned}\beta' = g(\beta) &= \beta^M + \binom{M}{1}\beta^{M-1}(1-\beta) + \dots \\ &+ \frac{1}{2}\binom{M}{M/2}\beta^{M/2}(1-\beta)^{M/2}.\end{aligned}$$

Next we study the step-wise reduction of each type of error probability.

Proposition 2: Consider an M-ary relay tree, where M is even. Suppose that we apply majority dominance as the fusion rule. Then,

$$1 \le \frac{\alpha_{k+1}}{\alpha_k^{M/2}} \le 2^{M-1}.$$

The proof is similar to that of Proposition 1 and it is omitted. Notice that the above result is only useful when $M \ge 4$. For the case where $M = 2$ (balanced binary relay trees), we have

$$\alpha_{k+1} = \alpha_k^2 + \alpha_k(1-\alpha_k) = \alpha_k$$

and

$$\beta_{k+1} = \beta_k^2 + \beta_k(1-\beta_k) = \beta_k;$$

that is, the Type I and II error probabilities remain the same after fusion. However, in [10], we have shown that with the unit-threshold likelihood-ratio test as the fusion rule at the relay nodes and the fusion center, the total error probability decays to 0 sub-exponentially with exponent $\sqrt{N}$.

From the above proposition, we derive upper and lower bounds for the Type I error probability at each level k.

Theorem 2: Consider an M-ary relay tree, where M is an even integer. Let $\lambda_M = M/2$. We have

$$\lambda_M^k(\log_2 \alpha_0^{-1} - (M-1)) \le \log_2 \alpha_k^{-1} \le \lambda_M^k \log_2 \alpha_0^{-1}.$$

The proof is similar to that of Theorem 1 and it is omitted. Similar with the case where M is odd, we can provide upper and lower bounds for the Type I error probability at the fusion center.

Corollary 2: Let $P_{F,N}$ be the Type I error probability at the fusion center of an M-ary relay tree, where M is even. We have

$$\begin{gathered}N^{\log_M \lambda_M}(\log_2 \alpha_0^{-1} - (M-1)) \le \\ \log_2 P_{F,N}^{-1} \le N^{\log_M \lambda_M} \log_2 \alpha_0^{-1}.\end{gathered}$$

Notice that the bounds in Corollaries 1 and 2 have the same form if we simply let $\lambda_M = \lfloor (M+1)/2 \rfloor$. In the next section, we use the bounds above to derive upper and lower bounds for the total error probability at the fusion center.

C. Bounds for Total Error Probability

Let π_0 and π_1 be the prior probabilities for the underlying hypotheses. In this section, we provide upper and lower bounds for the total error probability P_N at the fusion center. It is easy to see that

$$P_N = \pi_0 P_{F,N} + \pi_1 P_{M,N},$$

where $P_{F,N}$ and $P_{M,N}$ correspond to the Type I and II error probabilities at the fusion center. With the bounds for each type of error probability, we provide bounds for the total error probability as follows.

Theorem 3: Consider an M-ary relay tree, let $\lambda_m = \lfloor \frac{M+1}{2} \rfloor$. We have

$$\begin{gathered}N^{\log_M \lambda_M}(\log_2 \max\{\alpha_0, \beta_0\}^{-1} - (M-1)) \le \\ \log_2 P_N^{-1} \le N^{\log_M \lambda_M}(\pi_0 \log_2 \alpha_0^{-1} + \pi_1 \log_2 \beta_0^{-1}).\end{gathered}$$

Proof: From the definition of P_N, that is,

$$P_N = \pi_0 P_{F,N} + \pi_1 P_{M,N},$$

we have the following:

$$P_N \le \max\{P_{F,N}, P_{M,N}\}.$$

In addition, we know that α_k and β_k have the same recursion. Therefore, the maximum between the Type I and II error probability at the fusion center corresponds to the maximum at the leaf nodes. Hence, we have

$$N^{\log_M \lambda_M}(\log_2 \max\{\alpha_0, \beta_0\}^{-1} - (M-1)) \le \log_2 P_N^{-1}.$$

By the fact that $\log_2 x^{-1}$ is a convex function, we have

$$\log_2 P_N^{-1} \le (\pi_0 \log_2 P_{F,N}^{-1} + \pi_1 \log_2 P_{M,N}^{-1}).$$

Therefore, we have

$$\log_2 P_N^{-1} \le N^{\log_M \lambda_M}(\pi_0 \log_2 \alpha_0^{-1} + \pi_1 \log_2 \beta_0^{-1}).$$

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D. Asymptotic Rates

In this section, we study the decay rate of the error probabilities in the asymptotic regime. We show that in the case where M is even, the majority dominance rule is sub-optimal. We also compare our asymptotic results with those in [13].

First from Corollaries 1 and 2, we can easily derive the decay rate of the Type I and II error probabilities. For example, for the Type I error probability, we have the following.

Corollary 3: Consider an M-ary relay tree, let $\lambda_m = \lfloor \frac{M+1}{2} \rfloor$. If α_0 is fixed, then

$$\log_2 P_{F,N}^{-1} = \Theta(N^{\log_M \lambda_M}).$$

Proof: To analyze the asymptotic rate, we may assume that α_0 is sufficiently small, that is, $\alpha_0 < 2^{-(M-1)}$. In this case, the bounds in Corollaries 1 and 2 show that

$$\log_2 P_{F,N}^{-1} = \Theta(N^{\log_M \lambda_M}).$$

■

It is easy to show that $\log_M \lambda_M$ is monotone with respect to M. Moreover, as M goes to infinity, the limit of $\log_M \lambda_M$ is 1. That is to say, when M is very large, the decay is getting close to exponential. In terms of tree structures, when M is very large, the tree becomes short, and therefore achieves similar performance to that of bounded-height trees. From the fact that the Type I and II error probabilities follow the same recursion, it is easy to see that the Type II error probability at the fusion center decays to 0 with exponent $N^{\log_M(\lfloor \frac{M+1}{2} \rfloor)}$. Moreover, we can compute the decay rate of the total error probability.

Corollary 4: Suppose that (α_0, β_0) is fixed. Given any prior probabilities, we have

$$\log_2 P_N^{-1} = \Theta(N^{\log_M \lambda_M}).$$

For the total error probability at the fusion center, we have similar arguments with that for individual error probabilities. For large M, the decay of the total error probability is close to exponential.

Recall the results from [13], in which it is shown that, with any combination of fusion rules,

$$\log_2 P_N^{-1} = O(N^{\log_M \frac{(M+1)}{2}}). \quad (1)$$

The case where the relay nodes and the fusion center use the majority dominance rule (with random tie-breaking) to combine messages was considered in [13], in which case the decay rate of the total error probability is almost optimal. More precisely,

$$\log_2 P_N^{-1} = \Omega(N^{\log_M \lfloor \frac{(M+1)}{2} \rfloor}).$$

Our results for the odd M case is consistent with the results in [13]. The majority dominance rule in this case is essentially optimal in the sense of achieving the largest decay exponent.

$$\log_2 P_N^{-1} = \Theta(N^{\log_M \lfloor \frac{(M+1)}{2} \rfloor}) = \Theta(N^{\log_M \frac{M}{2}}). \quad (2)$$

However in the case where M is even, our results show that

$$\log_2 P_N^{-1} = O(N^{\log_M \lfloor \frac{(M+1)}{2} \rfloor}). \quad (3)$$

Compared with (1), which is the upper bound for $\log_2 P_N^{-1}$ in [13], our upper bound (3) is more tight in the case of even M and it has the exact same form with that of the lower bound; that is, we find the explicit decay rate (2) of the total error probability in this case. Second, the decay exponent shows that the majority dominance rule in this case is essentially sub-optimal in the sense of achieving the best decay exponent. For example, in the case of binary relay trees, the total error probability remains after fusion with the majority dominance rule. On the other hand, the likelihood-rate test with unit threshold achieves the decay exponent $\sqrt{N}$ [10].

IV. Concluding Remarks

We have studied the detection performance of M-ary relay trees. We have analyzed the step-wise reduction of the Type I and II error probabilities and derived upper and lower bounds for each error probability at the fusion center as functions of N. These bounds show that the total error probability converges to 0 sub-exponentially. In addition, we show that the majority dominance rule is optimal for M-ary relay trees when M is odd. However, the rule is sub-optimal when M is even.

Many interesting questions remain. For example, we have considered the impact of sensor failure and communication failure in balanced binary relay trees. How does failure affect detection performance in M-ary relay trees? More generally, what can we say about unbalanced relay trees or even more connected graph topologies?

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